*Mangalayatan Journal of Scientific and Industrial Research*

Man. J. Sci. Ind. Res.
Volume 1 (1) (2024), 24107

***Review Article***

# Applications of the Finite Element Method in Fluid Flow and Heat Transfer Analysis: A Review

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## Abstract

*Problems in heat transfer and fluid flow often involve solving governing equations with specific boundary conditions over a domain. These domains are typically irregular and complex, making exact analytical solutions challenging to obtain. The finite element method (FEM) is a powerful numerical approach for tackling these issues, as it can effectively discretize domains of any shape and size using a finite element mesh. This paper explores the use of FEM in analyzing heat transfer and fluid flow problems and discusses recent developments in this technique.*

***Keywords**: Finite Element Method, Fluid Flow, Heat Transfer, Mesh Generation.*

## 1. Introduction

Before the advent of personal computers, resolving design challenges and developing thoughtful or explanatory solutions often required considerable time. While these solutions often provided valuable insights into the behavior of specific systems, they were typically applicable to only a narrow set of problems. Since the late 1940s, the emergence of advanced computers has significantly accelerated the development and use of numerical methods. These methods are highly effective in addressing complex problems, including large-scale conditional systems, nonlinear phenomena, and intricate geometries that are often difficult to analyze or solve. For example, the governing equation of the fundamental two-dimensional heat conduction problem is:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \tag{1}$$

where u(x, y) is the temperature distribution in the Cartesian coordinates x, y, and is defined in a rectangular region $0 \leq x \leq a$, $0 \leq y \leq b$ , together with the boundary conditions:

$$u(0, y) = 0 \text{ and } 60 \leq y \leq b$$

$$u(x,0) = 0 \quad \text{and } u(x,b) = u_b \; 0 \leq x \leq a \tag{2}$$

$$u(x, y) = \frac{4u_0}{\pi} \sum_{k=1} \frac{1}{(2k+1)} \frac{\sinh \frac{(2k+1)}{a} \pi y}{\sinh \frac{(2k+1)}{a} \pi b} \sinh \frac{(2k+1)}{a} \pi x \tag{3}$$

Received: 28 Oct. 2024; Accepted: 15 Nov. 2024

Even though this articulation isn't basic, we still need to evaluate it using numerical techniques. Reworking the problem by considering several forms of discretization is appealing. By using a numerical approach, the ordinary term of a differential can be converted to a presumed discrete articulation. Because the problem is discretized, it only needs to be solved at a few specific locations within the region; appropriate additions may be used elsewhere in the district. In this way, the problem is reduced to a purely mathematical framework that only involves the basic number juggling exercises, which can therefore be solved using numerical techniques. To be precise, we have a variety of general discretization techniques, such as the Finite Element Method and the Finite Distinction Strategy. The Finite Difference Method is the simplest approach to use, but it requires high degree work regularities and is particularly effective on uniform networks.

The limited component technique is a common numerical process used in building evaluation to address beginning and limit esteem concerns.

## 2. History

The modern application of finite element methods began in structural engineering. The introduction of jet engines in the 1940s and the resulting increase in aircraft speeds prompted a shift from upswept to swept wing designs. The initial work in this area was by Hrennikoff [1], who established an analogy between discrete elements and corresponding sections of a continuous solid, applying it to aircraft structural design. Turner et al. [4] advanced the field by introducing the stiffness matrix for triangular elements based on displacement assumptions and describing the element assembly process using the direct stiffness method. Clough [7] later coined the term "finite element" in a paper discussing its application to plane elasticity problems.

Research into solving nonlinearity issues gained prominence, with Turner et al. [4] initiating the incremental technique for addressing geometrical nonlinearities and Martin [3] analyzing stability issues. Material nonlinearity, including plasticity and viscoelasticity, was explored by Gallagher et al. [8] and Zienkiewicz et al. [5], respectively. Melosh [8] utilized the principle of minimum potential energy, providing the first convergence proof in engineering literature. This work paved the way for the broader use of variational principles, significantly expanding FEM applications.

Zienkiewicz and Cheung [6] explored solutions to Poisson's equation, while Wilson and Nickell [2] studied transient heat conduction problems. The method also extended to biomedical engineering, addressing challenges involving geometric and material nonlinearities, a topic first investigated by Gould et al. [10].

## 3. Finite element method

The fundamental principle of the finite element method (FEM) involves dividing the domain into multiple subdomains, referred to as finite elements. These elements can vary in shape and properties, making them suitable for discretizing complex structures or those with mixed material characteristics. Moreover, they can precisely represent the geometry of domain boundaries, regardless of their complexity. To establish a "general-purpose" framework for addressing heat transfer and fluid flow problems, consider the following system of differential equations:

Au =fin Ω (4)

With the boundary conditions:

Bu =tin Γ (5)

where A is a system of governing equations defined in the domain Ω, B is a system of some boundary functions defined in the boundary Γ, and f, t are some functions. This system governs many applications in the engineering field. To find a solution to this system, apply the weight edresi dual method and yield:

$$\int_{\Omega} W_j (Au - f) d\Omega + \oint_{\Gamma} W_j (B\bar{u} - t) d\Gamma = 0 \quad (6)$$

where Wj(j = 1,…,n) are weighting functions and $\bar{u}$ is an approximation to the unknown u:

$$u = \bar{u} = \sum_{j=1}^{n} N_j u_j \quad (7)$$

in which $N_j$ are some basis functions and $u_j$ are the nodal values of the unknown.

Substituting equation (7) into equation (6), a system of equations can be obtained:

Ku = f (8)

where K is a square matrix, and u, f are some vectors.

The Galerkin version of FEM (GFEM) is defined when the weighting function in equation

$$W_j = N_j \quad (9)$$

This method leads to minimum errors and preserves the symmetry of matrix K, and it is the most frequently used version of FEM. Sometimes this method is also called the Bubnov-Galerkin methods (BGFEM). In recent year we have some other version of FEM like, Petrov-Galerkin finite element method (PGFEM), the finite volume method(FVM)etc.

## 4. Applications to Heat Transfer and Fluid Flow

### (i) Hybrid Schemes for Solving Nonlinear Convection-Diffusion and Compressible

**Viscous Flow Problems-**The viscosity and heat conduction coefficients of gases are relatively small, meaning that viscous dissipative effects are often treated as perturbations to the inviscid Euler system. This necessitates the use of an efficient numerical approach for solving inviscid flow problems. A hybrid scheme combining the finite volume method (FVM) and the finite element method (FEM) is proposed to address nonlinear convection-diffusion problems and compressible viscous flow. This approach employs a general cell-centered flux vector splitting FVM discretization for inviscid terms, while viscous terms are discretized using FEM on a triangular grid.

**Spatially Periodic Flows in Irregular Domains-**Two types of periodicities, translational and rotational, are considered based on the relative orientation of the modules. For complex flow geometries, periodic boundary-fitted grids are commonly employed over a representative module to simulate such flows.

To discretize the momentum and continuity equations in fluid flow, finite volume methods with non-staggered grids are frequently utilized.

### (ii) Acoustic Fluid-structure Interaction

Problems Two types of periodicities, translational and rotational, are considered based on the relative orientation of the modules. For complex flow geometries, periodic boundary-fitted grids are commonly employed over a representative module to simulate such flows. To discretize the momentum and continuity equations in fluid flow, finite volume methods with non-staggered grids are frequently utilized.

## 5. Conclusion

FEM of fersenormous flexibility in the treatment of nonlinearities, in homogeneities and anisotropy. The objective of this paper is to identify some trends in FEM and the irrelation to research in engineering. It is hoped that works from different disciplines, which common interest is finite element methods, can promote wide rawareness through out the finite element community of the latest developments in engineering and mathematics.

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