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Article

Anisotropic dark energy cosmological model of Bianchi type II with zero mass scalar fields

Rahul Kumar[1], Uday Raj Singh[2], Ajay Kumar Sharma[3], Rekha Kumari[4], Satendra Kumar Suman[5], Ram Naresh Singh Sisodiya[6], and Vinod Kumar Singh[7]

[1,2,5] Department of Mathematics, C.L Jain (P.G) College, Firozabad, (U.P), India
[3] Department of Mathematics, Mahatma Gandhi College of Science, Gadchandur, Dist-Chandrapur (M.S), India.
[4] Department of Mathematics, Government Raza (P.G) College Rampur, (U.P), India.
[6]Dept. of Mathematics, Sardar Patel Mahavidyalaya, Chandrapur, Maharashtra, India.
[7] Department of Mathematics, Chintamani College of Science, Pombhurna
Dist-Chandrapur (M.S), India.
Corresponding Author: vinodsingh.shibu@gmail.com

Abstract

In order to account for the observed phenomenon of rapid expansion inside the cosmos, modern cosmology has been forced to incorporate dark energy cosmological theories. Our investigation is driven by a desire to learn more about the nature of dark energy in the context of a locally rotationally symmetric (LRS) Bianchi type-II space-time that includes a mass less scalar field. Einstein's field equations are solved using the expansion law in an effort to create a deterministic model of the universe. In addition, a connection between the prospective metrics is used to achieve this aim. By using visualization, the physical implications of determining the model's dynamical cosmological parameters are made clear. Observations of the scalar field model show that there are fluctuations within the quintessence region, suggesting that the universe is undergoing a transition from a deceleration phase to an acceleration phase. The empirical findings of modern cosmology are consistent with this truth.

Keywords: Bianchi Type-II, EoS parameter, skewness parameter deceleration parameter, Ricci dark energy

1. Introduction

In recent years, there has been a surge of interest in anisotropic cosmological models due to experimental investigations into the isotropy of the Cosmic Microwave Background Radiation (CMBR), as well as speculations regarding the formation of helium during the early stages and various other phenomena. The interest in this topic has arisen from the recognition that the standard cosmological models, which accurately describe the present-day universe, fail to provide a comprehensive explanation for the early stages of its evolution. In contrast, inhomogeneous models offer a more physically realistic depiction of this phase. The study of homogeneous models is particularly appealing due of their mathematical simplicity. The Bianchi models, which exhibit spatial homogeneity and anisotropy, serve as an intermediary between the Friedman Robertson Walker (FRW) models and fully inhomogeneous and anisotropic universes. Consequently, they hold significant significance within the realm of contemporary cosmology. The lack of rigorous validity for the assumptions of spherical symmetry and isotropy near the Big Bang singularity is the cause of this phenomenon. This statement highlights the motivation to do research aimed at obtaining precise anisotropic solutions for Einstein's field equations, as well as developing a cosmologically viable physical model for the

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universe. When examining the nature of cosmic solutions within relativistic theories of gravity, it is customary to employ the energy momentum tensor of matter in a configuration that resembles that of a perfect fluid. Homogeneous cosmologies including a perfect fluid that adheres to a particular equation of state are extensively employed in the examination of many characteristics of cosmological models. The investigation of Bianchi type-II space time models has been extensively conducted in order to simplify and depict the macroscopic dynamics of the observable cosmos.

The review done by Pradhan et al. (2009) focused on the assessment of locally rotationally symmetric (LRS) Bianchi type II string cosmological models portrayed by the presence of an optimal liquid scattering of issue. Saha (2011) gave proof that the presence of a non-zero off-corner to corner part in the Einstein tensor forces a few thorough constraints on the choice of issue dissemination. The forced limits are tough. The review led by Reddy and Kumar (2013) inspected the LRS Bianchi Type-II space-time with the consideration of an ideal liquid source, using the f(R,T) gravity structure. In their review, Ghate et al. (2015) explored the qualities of Bianchi type-IX cosmological models inside the structure of general relativity. In particular, they analyzed the way of behaving of these models within the sight of dim energy, while thinking about a variable condition of state (EoS) boundary. Borkar and Ashtankar (2016) determined an answer for the LRS Bianchi type-II space-time, integrating a string gooey liquid and an attractive field, by the utilization of the Stylist's field conditions inside the structure of the self-creation hypothesis of attractive energy. In a review directed by Agrawal (2017), the examination zeroed in on the investigation of Einstein's field conditions inside the setting of locally-rotationally-symmetric Bianchi type-II space-time. The concentrate additionally investigated the impacts of the presence of solid liquid matter and a variable cosmological element. The trial was led within the sight of a cosmological steady. Reddy et al. (2018) directed a review that zeroed in on the examination of the Bianchi type-II dim energy cosmological model, consolidating the gravitational field related with zero mass scalar fields. Singh and Singh (2019) directed an investigation into the anisotropic Locally Rotationally Symmetric (LRS) Bianchi type-I metric, consolidating dim energy inside the structure of the changed $f(R,T)$ hypothesis of gravity. Inside the given system, the image R signifies the Ricci scalar, while the image T addresses the hint of the pressure energy-force tensor. In their review, Chundawat and Mehta (2020) analyzed the Hoyle - Narlikar C-field cosmology inside the structure of the LRS Bianchi type II model, consolidating a period shifting cosmological steady (t) and considering a barotropic ideal liquid dispersion. Banerjee et al. (2021) directed a concentrate on a cosmological model of Bianchi type-II, which displayed spatial homogeneity and neighborhood rotational evenness. The model was inspected under the consolidated impacts of shear and mass consistency. In their review, Koussour et al. (2022) led an assessment of the anisotropic locally rotationally symmetric (LRS) Bianchi type-I space-time inside the setting of the recently proposed f(Q) gravity, where Q addresses the non-metricity scalar. In the domain of $f(T)$ gravity, the review led by Qazi et al. (2023) zeroed in on the assessment of exact cosmological arrangements of the Bianchi-type V model. This examination was completed by utilizing conformal vector fields (CVFs) that fulfill energy conditions.

2. Exact equations of fundamental fields: In the context of a spatially homogeneous Bianchi type-II model, the metric employed is the Locally Rotationally Symmetric (LRS) metric.

$$ds^2 = -dt^2 + A^2(dx - zdy)^2 + B^2(dy^2 + dz^2) \tag{1}$$

A and B represent functions that are exclusively dependent on cosmic time, denoted by the variable $'t'$.

The inclusion of anisotropic dark energy (DE) fluid and scalar meson fields results in the formulation of a novel system of equations governing the Einstein field.

$$R_{ij} - \frac{1}{2} g_{ij} R = -T_{ij} - \left(V_{,i} V_{,j} - \frac{1}{2} g_{ij} V_k V^k \right) \quad (2)$$

By utilizing Equation (6), we may express the explicit Einstein field equations (2) for the metric. (1):

$$g_{ij} V_{;ij} = 0 \quad (3)$$

In mathematical notation, the use of a semicolon typically denotes the covariant derivative, while a comma is commonly employed to represent the conventional derivative. Modern cosmological research has provided confirmation of the acceleration of the cosmos, which is ascribed to the presence of dark energy. Certain cosmological expansion theories, known as quintessence theories, attribute the involvement of dynamic scalar fields. A scalar field with a negative equation of state was developed as a substitute for the cosmological constant in a more comprehensive context. As a result of this, we have been contemplating the concept commonly referred to as the fundamental element. In mathematical notation, the use of a semi-colon is generally employed to represent the covariant derivative, while a comma is typically used to describe the ordinary derivative. Contemporary cosmological measurements provide empirical evidence supporting the phenomenon of cosmic acceleration, which is postulated to be attributed to the presence of dark energy. Theories of quintessence propose that the cosmic expansion is driven by dynamical scalar fields. A substitution of the cosmological constant was made by implementing a scalar field with a more broadly negative equation of state. This observation has prompted us to contemplate the quintessential entity.

The scalar field denoted by V is characterised by having zero mass, and it is required to satisfy the Klein-Gordon equation.

$$p_\Lambda = \rho_\Lambda \omega_\Lambda \quad (4)$$

The fluid's equation of state (EoS) parameter is represented by the symbol ω, whereas $\omega_x, \omega_y, \omega_z$ denote the EoS parameters along the x, y and z axes, respectively. The expression for the parameterization of the energy momentum tensor of the dark energy fluid can be given as.

$$T_i^j = diag[-1, \omega_\Lambda, (\omega_\Lambda + \delta), (\omega_\Lambda + \delta)]\rho_\Lambda \quad (5)$$

Explicit Einstein field equations (2) for the metric (1) with the help of Eq. (5) can be written as

$$\left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} - \frac{1}{4}\frac{A^2}{B^4} - \frac{\dot{V}^2}{2} = \rho_\Lambda \quad (6)$$

$$2\frac{\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 - \frac{3}{4}\frac{A^2}{B^4} + \frac{\dot{V}^2}{2} = -\omega_\Lambda \rho_\Lambda \quad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{1}{4}\frac{A^2}{B^4} + \frac{\dot{V}^2}{2} = -(\omega_\Lambda + \delta)\rho_\Lambda \quad (8)$$

The Klein-Gordon equation, which characterizes the dynamics of a scalar field in the presence of a specified metric (1), can be mathematically represented as follows:

$$\ddot{V} + \dot{V}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) = 0 \tag{9}$$

$$\rho_\Lambda + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)(1 + \omega_\Lambda)\rho_\Lambda = 0 \tag{10}$$

3. Solution of the field equations: In this analysis, we address the resolution of the field equations (6)-(10), which constitute a set of four distinct equations. Equation (10) is a direct outcome of equations (6)-(9). These equations involve six variables, namely A, B, V, ρ_Λ and ω_Λ, which are considered as unknowns in the system. Therefore, it is necessary to introduce two additional conditions in order to obtain a definitive solution. The physically viable conditions that we employ are as follows:

$$A = B^m \tag{11}$$

Let m denote a positive constant, with the exception of the value -1, which represents the anisotropy of the space-time.

In this analysis, we shall explore the notion of the average scale factor.

The positive constant m, with the condition $m \neq -1$ is responsible for the anisotropy of the space-time.

In this analysis, we shall investigate the notion of the average scale factor, denoted as

$$a(t) = \begin{Bmatrix} (n\alpha t + \beta)^{\frac{1}{n}} & n \neq 0 \\ \delta e^{\alpha t} & n = 0 \end{Bmatrix} \tag{12}$$

Furthermore, we have also

$$V = a^3 = AB^2 \tag{13}$$

For any non-zero value of n, the equations (11), (12), and (13) can be solved to provide the solution.

$$(n\alpha t + \beta)^{\frac{3}{n}} = B^m\, B^2 = B^{m+2}$$

$$B^{m+2} = (n\alpha t + \beta)^{\frac{3}{n}}$$

$$B = \quad (n\alpha t + \beta)^{\frac{3}{n(m+2)}} \tag{14}$$

$$A = (n\alpha t + \beta)^{\frac{3m}{n(m+2)}} \tag{15}$$

By carefully selecting appropriate coordinates and constants, the metric (1) can be expressed using equations (14) and (15) as

$$ds^2 = -dt^2 + \left[(n\alpha t + \beta)^{\frac{3m}{n(m+2)}}\right](dx - zdy)^2 + \left[(n\alpha t + \beta)^{\frac{3}{n(m+2)}}\right](dy^2 + dz^2) \tag{16}$$

By utilizing equations (9), (14), and (15), the zero mass scalar field may be derived.

$$V = V_0 \int (n\alpha t + \beta)^{-\frac{3}{n}} \, \mathrm{dt} + c_2 \tag{17}$$

The constant of integration c_2 can be assigned a value of zero.

$$V = V_0 \int (n\alpha t + \beta)^{-\frac{3}{n}} \, \mathrm{dt} \tag{18}$$

$$V = V_0 \frac{(n\alpha t+\beta)^{\frac{n-3}{n}}}{\alpha(n-3)}, n \neq 3 \tag{19}$$

The LRS Bianchi type-II DE model ($n \neq 0$) in the presence of zero mass scalar fields can be represented by Equation (19) in conjunction with Equation (16).

For $n = 0$, Solving equations (11),(12) and (13), we get

$$B = (\delta e^{\alpha t})^{\frac{3}{m+2}} \tag{20}$$

$$A = (\delta e^{\alpha t})^{\frac{3m}{m+2}} \tag{21}$$

By appropriately selecting coordinates and constants, the metric (1) can be expressed utilizing

$$ds^2 = -dt^2 + (\delta e^{\alpha t})^{\frac{3}{m+2}}(dx - zdy)^2 + (\delta e^{\alpha t})^{\frac{3}{m+2}}(dy^2 + dz^2) \tag{22}$$

By utilizing equations (9) and (22), the zero mass scalar field may be derived.

$$V = V_0 \int e^{-3\alpha t} \, \mathrm{dt} + c_2 \tag{23}$$

Where the constant of integration c_2 can be set equal to zero.

$$V = V_0 \int e^{-3\alpha t} \, \mathrm{dt} \tag{24}$$

$$V = -\frac{V_0}{3\alpha} e^{-3\alpha t} \tag{25}$$

The LRS Bianchi type-II DE model (n=0) in the presence of zero mass scalar fields can be represented by Equation (25) in conjunction with Equation (22).

4. Some Physical properties of Model:

The cosmological parameters that are pertinent to the physical examination of the model (6-10) are defined and ascertained as follows:

4.1. The spatial volume:

$$V = AB^2 = \begin{Bmatrix} (n\alpha t + \beta)^{\frac{3}{n}} & n \neq 0 \\ \delta^3 e^{3\alpha t} & n = 0 \end{Bmatrix} \tag{26}$$

4.2 Average Hubble parameter:

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) = \begin{Bmatrix} \frac{\alpha}{n\alpha t+\beta} & n \neq 0 \\ \alpha & n = 0 \end{Bmatrix} \tag{27}$$

4.3 The scalar expansion: $\theta = 3H = \begin{Bmatrix} \frac{3\alpha}{n\alpha t+\beta} & n \neq 0 \\ 3\alpha & n = 0 \end{Bmatrix}$ (28)

4.4. The shear scalar:

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 = \begin{Bmatrix} \frac{\alpha^2(m-1)^2}{(n\alpha t+\beta)^2(m+2)^2} & n \neq 0 \\ \frac{\alpha(m-1)^2}{(m+2)^2} & n = 0 \end{Bmatrix} \tag{29}$$

$$\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{9(m+2)^2} \tag{30}$$

4.5 Mean anisotropy parameter:

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{2(m-1)^2}{(m+2)^2}, for\ all\ n \tag{31}$$

4.6 The deceleration parameter:

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) = \begin{Bmatrix} n-1 & n \neq 0 \\ -1 & n = 0 \end{Bmatrix} \tag{32}$$

4.7 Energy Density:

$$\rho_\Lambda = \begin{Bmatrix} \frac{9\alpha^2(2m+1)}{(n\alpha t+\beta)^2(m+2)^2} - \frac{1}{4}(n\alpha t+\beta)^{\frac{6(m-2)}{n(m+2)}} - \frac{{V_0}^2}{2}(n\alpha t+\beta)^{-\frac{6}{n}} & n \neq 0 \\ \frac{9(2m+1)\alpha^2}{(m+2)^2} - \frac{1}{4}(\delta e^{\alpha t})^{\frac{6(m-2)}{m+2}} - \frac{{V_0}^2}{2}e^{-6\alpha t} & n = 0 \end{Bmatrix} \tag{33}$$

4.8 EoS parameter:

$$\omega_\Lambda = \begin{Bmatrix} -\frac{\frac{3\alpha^2\{9-2n(m+2)\}}{(m+2)^2(n\alpha t+\beta)^2} - \frac{3}{4}(n\alpha t+\beta)^{\frac{6(m-2)}{n(m+2)}} + \frac{{V_0}^2}{2}(n\alpha t+\beta)^{-\frac{6}{n}}}{\frac{9\alpha^2(2m+1)}{(n\alpha t+\beta)^2(m+2)^2} - \frac{1}{4}(n\alpha t+\beta)^{\frac{6(m-2)}{n(m+2)}} - \frac{{V_0}^2}{2}(n\alpha t+\beta)^{-\frac{6}{n}}} & n \neq 0 \\ -\frac{\frac{27\alpha^2}{(m+2)^2} - \frac{3}{4}(\delta e^{\alpha t})^{\frac{6(m-2)}{(m+2)}} + \frac{{V_0}^2}{2}e^{-6\alpha t}}{\frac{9(2m+1)\alpha^2}{(m+2)^2} - \frac{1}{4}(\delta e^{\alpha t})^{\frac{6(m-2)}{m+2}} - \frac{{V_0}^2}{2}e^{-6\alpha t}} & n = 0 \end{Bmatrix} \tag{34}$$

4.9 Skewness parameter:

$$\delta = \begin{Bmatrix} -\frac{\frac{3\alpha^2[n(m+2)(m-1)-2n(m+2)-3m^2+6]}{(m+2)^2(n\alpha t+\beta)^2} - \frac{9\alpha^2 m}{(m+2)(n\alpha t+\beta)} - (n\alpha t+\beta)^{\frac{6m(m-2)}{n(m+2)}}}{\frac{9\alpha^2(2m+1)}{(n\alpha t+\beta)^2(m+2)^2} - \frac{1}{4}(n\alpha t+\beta)^{\frac{6(m-2)}{n(m+2)}} - \frac{{V_0}^2}{2}(n\alpha t+\beta)^{-\frac{6}{n}}} & n \neq 0 \\ \frac{\frac{9\alpha(1+3\alpha-\alpha m^2+\alpha m)}{(m+2)^2} - (\delta e^{\alpha t})^{\frac{6(m-2)}{(m+2)}}}{\frac{9(2m+1)\alpha^2}{(m+2)^2} - \frac{1}{4}(\delta e^{\alpha t})^{\frac{6(m-2)}{m+2}} - \frac{{V_0}^2}{2}e^{-6\alpha t}} & n = 0 \end{Bmatrix} \tag{35}$$

4.10 Jerk Parameter: $j = q + 2q^2 - \frac{\dot{q}}{H}$

$$j = \begin{Bmatrix} (n-1) + 2(n-1)^2 & n \neq 0 \\ 1 & n = 0 \end{Bmatrix}$$

$$j = \begin{cases} 2n^2 - 3n + 1 & n \neq 0 \\ 1 & n = 0 \end{cases}$$

$$j = \begin{cases} (2n-1)(n-1) & n \neq 0 \\ 1 & n = 0 \end{cases} \tag{36}$$

4.11 $\{r, s\}$ parameter:

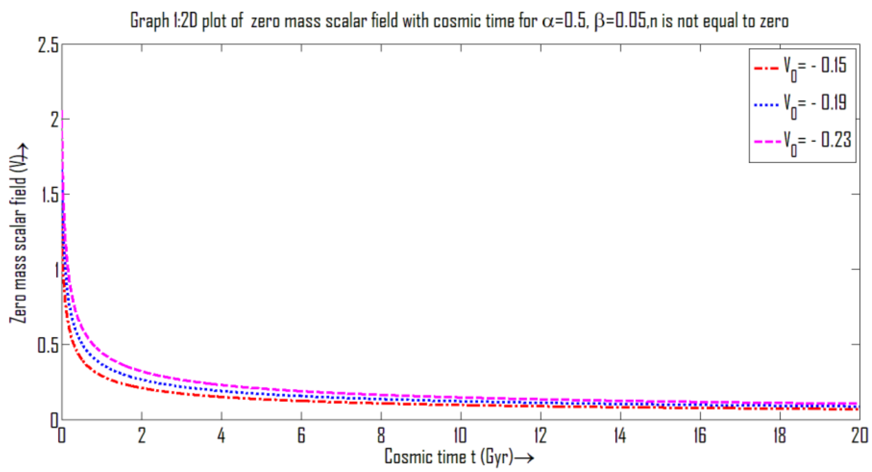
$$r = \frac{1}{H^3}\frac{\dddot{a}}{a} = \frac{1}{\alpha^3}\frac{\delta\alpha^3 e^{\alpha t}}{\delta e^{\alpha t}} = 1$$

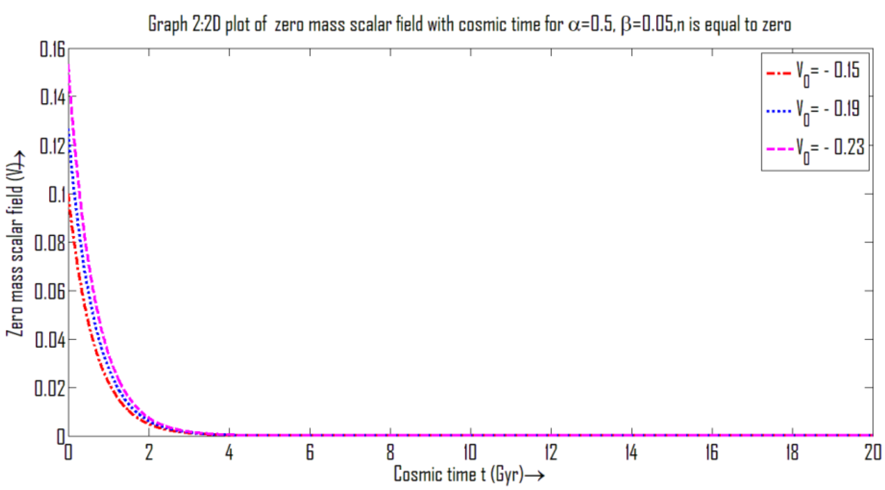
$$r = \begin{cases} \frac{(2n-1)(n-1)}{\alpha(n\alpha t+\beta)} & n \neq 0 \\ 1 & n = 0 \end{cases} \tag{37}$$

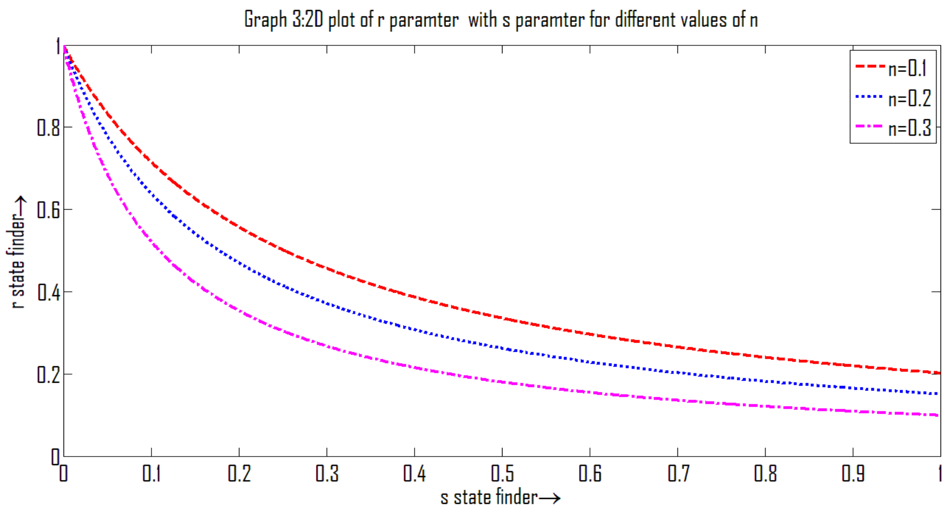
$$s = \begin{cases} \frac{2}{3}\frac{(2n-1)(n-1)-\alpha(n\alpha t+\beta)}{n-2} & n \neq 0, n \neq 2 \\ 1 & n = 0 \end{cases} \tag{38}$$

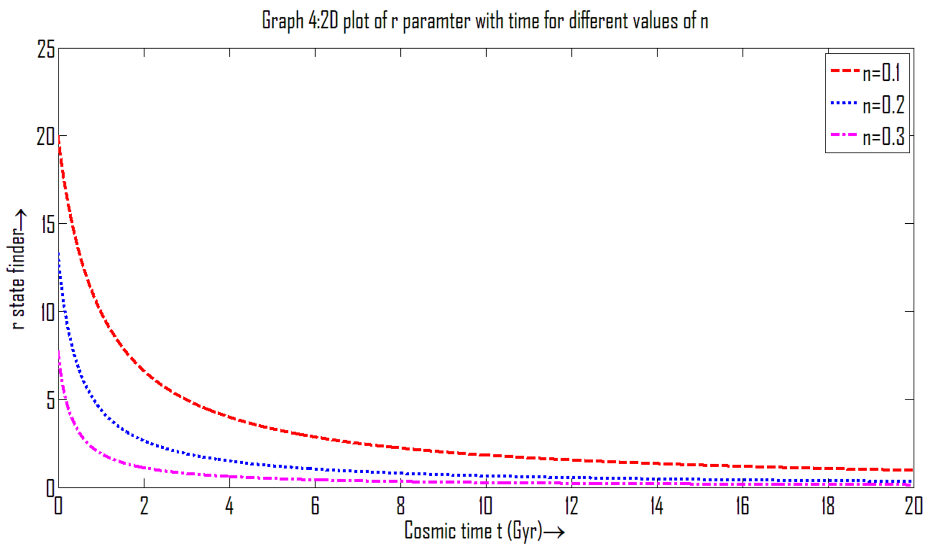
$$r = \frac{2(2n-1)(n-1)}{2(2n-1)(n-1)-3(n-2)s}, n \neq 0, n \neq 1, \frac{1}{2} \tag{39}$$

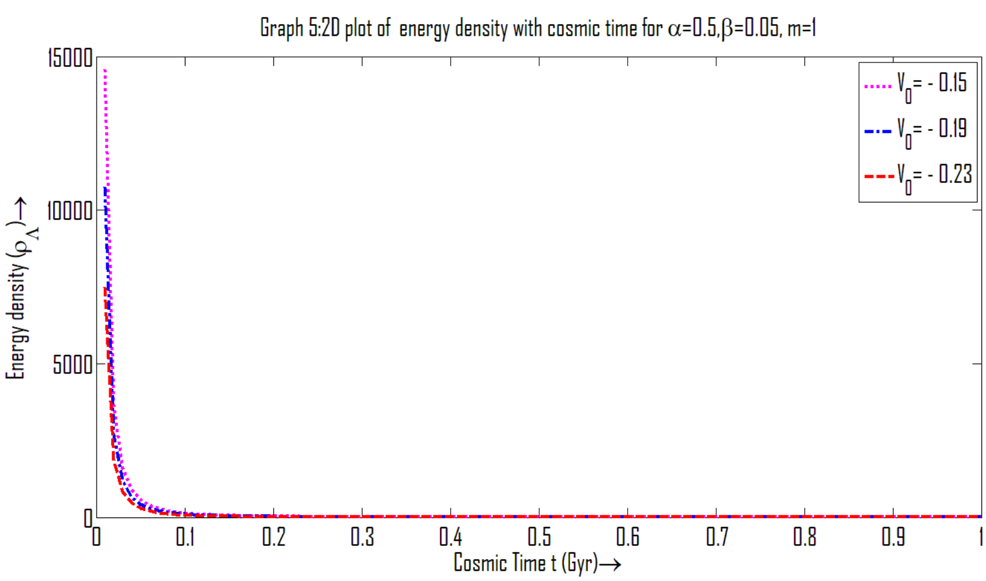
5. Graphical explanation of cosmological parameters:



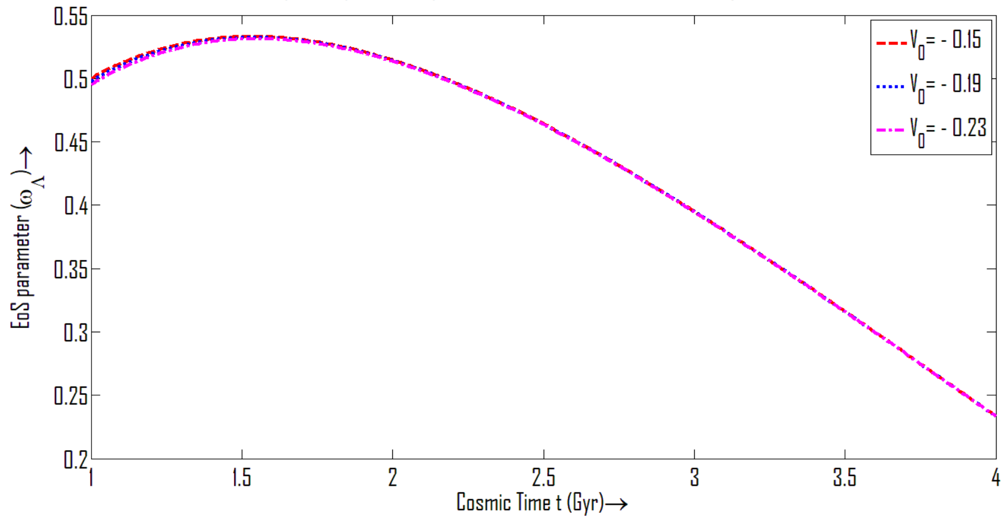



Graph 3:2D plot of r paramter with s paramter for different values of n
r state finder→
s state finder→
n=0.1
n=0.2
n=0.3

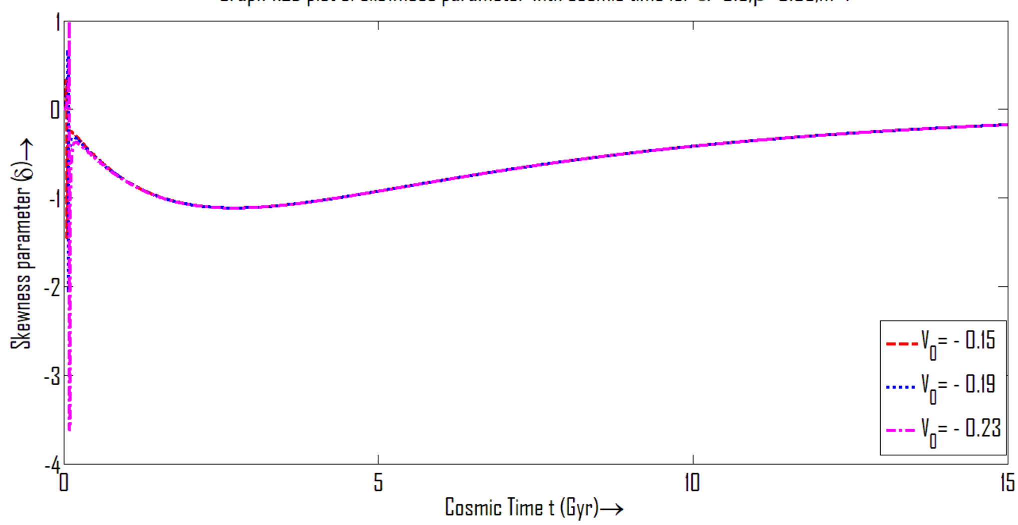

Graph 4:2D plot of r paramter with time for different values of n
r state finder→
Cosmic time t (Gyr)→
n=0.1
n=0.2
n=0.3


Graph 5:2D plot of energy density with cosmic time for α=0.5,β=0.05, m=1
Energy density (ρ_Λ)→
Cosmic Time t (Gyr)→
V_0= - 0.15
V_0= - 0.19
V_0= - 0.23

Graph 6:2D plot of EoS parameter with cosmic time for α=0.5,β=0.05, m=1



Graph 7:2D plot of skewness parameter with cosmic time for α=0.5,β=0.05,m=1



According to Equation (26), it can be inferred that the spatial volume is finite. This implies that the universe commences its evolution with a specific finite volume at $t = 0$ and thereafter increases as cosmic time progresses. In the limit when t approaches infinity and is not equal to zero, the expansion scalar diverges to infinity, but at $t = 0$ it converges to a finite value. The expansion scalar is finite when the value of n is equal to zero. Based on Equations (14), (15), (20), and (21), it can be observed that the spatial scale factors remain non-zero for all values of time (t), indicating the absence of singularity in the model. The scalar expansion and shear scalar approach zero as t approaches infinity, and they exhibit divergence at $t = 0$. Based on Equations (28) and (30), it can be shown that the mean anisotropy parameter A_m remains constant. Additionally, the limit as t approaches infinity of $\frac{\sigma^2}{\theta^2}$, denoted as $\lim_{t \to \infty} \frac{\sigma^2}{\theta^2} \; (\neq 0)$, also remains constant. Consequently, it can be concluded that the model exhibits anisotropy throughout the whole evolution of the universe, except at $m = 1$, when the model does not approach isotropy. The mean anisotropy parameter remains constant during the evolution of the universe for values of m not equal to -2. However, with m equal to 2, the mean anisotropy parameter approaches infinity. By examining equation (27), it becomes evident that the average Hubble parameter tends towards zero as time approaches infinity. As the limit of t approaches infinity and n is not equal to zero.

The behavior of the scalar field for $n \neq 0$ and $n = 0$ is seen in graphs (1) and (2) correspondingly. It can be observed that an increase in the initial velocity, denoted as V_0, leads to a decrease in the scalar field within the model. Furthermore, it has been observed that the impact of the fluctuation of V_0 becomes insignificant throughout later stages. The graph (3) demonstrates the variation in the r state finder with the s state finder for different values of n. It has been found that the value of the state finder parameter, denoted as r, falls when the values of the state finder parameter s and the parameter n grow. The graph (4) illustrates the relationship between the variation in the r state finder and cosmic time for different values of n. A drop in the state finder parameter, denoted as r, is noticed as cosmic time and the parameter n grow. The graph labelled as (5) illustrates the characteristics of dark energy density. It is evident that the dark energy density has a positive value and experiences a decline throughout cosmic time across various V_0 values. In the early stages, an increase in the scalar field corresponds to a decrease in the density of dark energy. However, as time progresses, the influence of the scalar field on the density of dark energy becomes nearly insignificant. Graph (6) illustrates the relationship between the skewness parameter and cosmic time t, demonstrating the anisotropic nature of dark energy as it evolves throughout the course of the universe. It is noteworthy to observe that when cosmic time reaches a sufficiently great magnitude, the skewness parameter becomes zero. Furthermore, it is seen that the scalar field does not have a substantial impact on the skewness parameter. The graph labeled as (7) illustrates the temporal evolution of the equation of state (EoS) parameter. It is evident that the dark energy model exhibits variability within the quintessence region, and it becomes apparent that the influence of the scalar field is quite insignificant during later periods. We are currently in the process of acquiring a quintessence scalar field model.

6. Concluding Remarks: This study explores a dark energy model with anisotropy, specifically focusing on its coupling with scalar fields of zero mass in a space-time characterized by the LRS Bianchi type-II metric. This study presents a precise solution to the Einstein field equations by establishing a connection between metric potentials and a hybrid expansion for the average scale factor of the universe. In this study, a differential equation (DE) model is proposed and subsequently utilized to establish the dynamical cosmological parameters associated with the model. The obtained results are then analyzed and their physical implications are explored using graphical illustrations. The observation reveals that our model undergoes a progression from its beginning epoch, expanding and undergoing a change from a decelerated phase to an accelerated stage. The observed skewness parameter indicates that the presence of dark energy maintains an anisotropic nature throughout the entire cosmic development, eventually diminishing over an extended period of time. The density of dark energy is positive and exhibits a decreasing trend as cosmic time progresses. The scalar field model, which we are considering, exhibits variations consistently inside the quintessence region. Therefore, we have derived a cosmological model that aligns with contemporary cosmological observations.

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