Statistical considerations in superposed epoch analysis and its applications in space research

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Abstract

Superposed epoch analysis is often used to demonstrate an effect or a periodicity. This method of analysis was originally proposed by Chree (Some phenomena of sunspots and of terrestrial magnetism at Kew observatory. Philosophical Transactions Royal Society London Series A 212, 75) and applied for studying the time variation of geophysical data. In its first application, Chree reported a 27-day periodicity (recurrence tendency) in geomagnetic data. Since then this method of analysis is being used in several disciplines either for testing the relationship between two diverse phenomena or to search for periodicities in the data. In addition to cosmic ray physics, various fields of research in which this method of analysis is often used include solar, magnetospheric, heliospheric, ionospheric and atmospheric physics as well as astrophysics and meteorology/climatology etc. Although a powerful method, an appropriate procedure to test the level of significance (statistical reality) of the obtained results is still lacking. This is highly desirable as, in the absence of a suitable test, a spurious/undesirable signal may appear as a ‘genuine’ effect.

This paper describes two techniques with application, one based on $t$-test and other on $F$-test, to test the significance level of results obtained on the basis of superposed epoch (Chree) analysis. Since most of the data acquired in atmosphere and space show a solar cycle variation, an appropriate procedure is also described for the data transformation (removal of solar cycle variation) before subjected to test a ‘genuine’ effect. To highlight the necessity for removal of solar cycle effect, a comparison of the results of significance test, before and after the data transformation, is also presented. Although these techniques are applicable to solar/astrophysical/heliospheric/magnetospheric/ionospheric/atmospheric/meteorological data, the procedure is illustrated using cosmic ray data. Details of the procedure to test the Forbush-decrease effect in cosmic ray intensity observed due to passage of interplanetary shocks, are discussed. Test results by two statistical procedures, one based on $t$-test and another based on $F$-test, are also compared in this paper. The effectiveness of interplanetary shocks on transient modulation of cosmic rays is studied and tested by both techniques. Both the techniques lead to similar conclusions. It is demonstrated, using both the techniques, that the effect of interplanetary shocks on transient modulation of cosmic ray intensity is statistically significant only due to the shocks whose sudden commencement amplitude exceeds certain limits.

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1. Introduction

Superposed epoch analysis has been employed very profitably for many solar-terrestrial, space and atmospheric physics problems. Although this method of analysis is being used since several decades (Chree, 1912, 1913), a suitable procedure to determine the statistical significance of the results is still required. Forbush and coworkers (Forbush et al., 1982, 1983) have emphasized the need for proper statistical method for assessing Chree analysis results and developed a statistical procedure for evaluating the quasi-persistence of the results obtained by superposed epoch analysis. Test of significance procedure for any deviation (effect) in the parameters under study, based on Chree analysis, have been given earlier too (Haung and Lee, 1975; Hoyt and Schatten, 1997). But, rarely any of these have been exploited in subsequent work.

Superposed epoch analysis has been utilized by several groups for investigating solar rotation and solar oscillations (Greaves et al., 1980), periodic and non-periodic perturbations in geomagnetosphere (Greaves and Newton, 1929; Iyemori and Rao, 1996; Badruddin, 1998; Kamide et al., 1998), solar wind, interplanetary magnetic field and cosmic ray variations in the heliosphere (Mayer and Simpson, 1954; Badruddin, 2002), variation of temperature profile and wind characteristics in the atmosphere (Pudovkin et al., 1997), changes in ozone contents in the stratosphere (Lastovicka and Mlch, 1999), cloud behavior in the troposphere (Todd and Kniveton, 2001) etc. Some other applications of Chree analysis, for example, geoeffectiveness of heliospheric current sheet (Mendoza and Perez-Enriquez, 1995) and the existence of load and dynamo region just inside the magnetosphere (Phan and Paschman, 1996) were substantiated by superposed epoch analysis. Following Forbush decreases, changes in atmospheric circulation and tropospheric variations (Gabis and Troshichev, 2000) were examined using Chree analysis. This method was employed—to look for any possible link—between cosmic ray flux and rainfall/precipitation level (Stozhkov et al., 1995), between cosmic ray flux and air–earth current density (Engfer and Tinsley, 1999) and between cosmic ray flux and sun-shine records (Palle and Butler, 2001). Solar-meteorological relations were studied by using superposed plots of sunspot numbers, geomagnetic aa index, temperature and rain fall (Baranyi and Ludmany, 1992). Ionospheric absorption associated with interplanetary magnetic clouds (Marcz, 1992) and ionization effects in the ionospheric D-region at mid-latitudes due to Forbush decreases (Satori, 1991) have been demonstrated with the help of this analysis. Apart from establishing 27-days recurrence tendency of cosmic ray intensity, Chree method of superposed epoch analysis has been widely used for the study of the effects of various solar/interplanetary phenomena on the cosmic ray intensity modulation, for example high speed streams (Badruddin, 1997), corotating interaction regions (Duggal et al., 1981), magnetic clouds (Badruddin et al., 1986; Zhang and Burlaga, 1988; Lockwood et al., 1991; Ananth and Venkatesan, 1993), interplanetary shocks (Badruddin, 2002). Changes of solar wind parameters (density, temperature, velocity, magnetic field) in the vicinity of heliospheric current sheet (Borrini et al., 1981) and one of remarkable findings of SWICS experiment on Ulysses-an anti-correlation between solar wind speed and ionization temperature and a direct correlation between ionization temperature and the ratio of the abundances of easily ionized magnesium to oxygen was obtained using superposed epoch analysis plot (Geiss et al., 1995). Solar activity shows several periodicities, but with irregular aspects. To analyze the irregular aspect of solar activity, superposed epoch results of fractal dimensionalities (a parameter for quantitatively describing the characteristics of irregular time series) and bending points deduced from sunspots number, 10.7 cm radio flux, and Fe XIV coronal emission before and after solar maximum as well as before and after solar minimum were plotted to see whether fractal dimensions and bending points change in concert with solar cycles or not (Watari, 1995). In addition to various disciplines of solar-terrestrial, space and atmospheric physics (magnetospheric/heliospheric/atmospheric/ionospheric physics), Chree analysis was employed in the field of chronobiology/space biology too. For example, increase in myocardial infarction after, in human heart rate during, and in blood pressure before the southward turn of north-south component of interplanetary magnetic field (Breus et al., 1995), strong geomagnetic perturbations effects and meteorological effects of myocardial infarctions, brain stroke, heart rhythm disturbances and sudden death have been investigated (Villoresi et al., 1994) with the help of superposed epoch analysis. In most of these wide range of applications conclusions have been drawn without testing statistical significance of the results.
2. Method

Superposed epoch (Chree) analysis has been extensively used to demonstrate an effect. In particular, the effect of solar phenomena such as sunspots/solar flares/coronal holes/disappearing filaments/prominences/coronal mass ejections and their interplanetary manifestations (shocks, magnetic clouds) on solar wind/geomagnetic activity/cosmic ray intensity/space and atmospheric weather/ionspheric absorption/ozone concentration etc., have been widely studied by use of this method. However, in most of studies error bars are not often given, or if they are shown, it is uncertain how the bars are obtained. In addition to estimating the standard error of the mean and plotting bars on the superposed epoch results, to our knowledge, other methods for testing the significance of any deviation (effect) observed on the basis of Chree analysis, in the parameters under study, have been given by Haung and Lee (1975), Forbush et al. (1982) and Hoyt and Schatten (1997). But rarely any of these have been exploited in subsequent work involving Chree method of analysis. The reason might be; either the described procedure is a bit complicated, incomplete or inappropriate. We have not done rigorous exercise to comment over it, however, we suspect that (superimposed) solar cycle effect observed in most of the heliospheric/magnetospheric/ionspheric/atomicpheric data, sometimes even larger than the ‘signal’ amplitude, may lead to unexpected conclusion and hamper the effort. For example, while studying the effect of a certain solar phenomena on transient modulation in cosmic ray intensity (Forbush decrease), one uses a data where the amplitude of Forbush decreases is usually ~2−10% while solar cycle variation almost is up to ~20−30%. Thus it is essential to correct (transform) the data (to remove solar cycle effect), before subjecting the data to significance test for genuine signal (Forbush effect). Forbush decreases are depressions in cosmic ray intensity with sudden onset, reaching minimum depression within about a day and have a more gradual recovery typically lasting several days. These decreases are caused by magnetic field and flow configurations that propagate away from the Sun (Badruddin, 2002).

2.1. Removal of solar cycle effect

Before evaluating the statistical significance of the ‘effect of interest’ (e.g. Forbush decrease), the data is transferred (corrected) to remove the major effects other than one under study. In our data (cosmic ray counts) the other major effect, that has to be removed, is due to solar cycle variation. For this purpose, we have adopted a procedure for data transformation without affecting the sample average and the ‘genuine’ effect (signal). This has been done by transforming (shifting) the data sets of each epoch to the level of sample mean.

This has been done as described below. Let $\bar{X}_i$ measurement of $i$th population is denoted by $X_{ij}$, $J$ being the number of observations in each sample and $I$ being the number of populations (epochs) being analyzed. The total data consists of $IJ$ observations. If we take (individual) sample means to be $\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_I$, then

$$\bar{X}_i = \frac{\sum_{j=1}^{J} X_{ij}}{J},$$

where $i = 1, 2, 3, \ldots, I$.

The dot in place of second subscript signifies that we have added over all values of that subscript while holding the other subscript value fixed, and the horizontal bar indicates division by $J$ to obtain an average. The average of all $IJ$ observations i.e. (grand) sample mean

$$\bar{X} = \frac{\sum_{i=1}^{I} \sum_{j=1}^{J} X_{ij}}{IJ}.$$  

The value ($\bar{X} - \bar{X}_i$) is then added/(subtracted) to/(from) each values of $i$th epoch. In this way, we are able to remove the solar cycle effect without altering the (grand) sample mean and ‘genuine’ signal in each epoch. Then a formal test procedure is applied (also see, Haung and Lee, 1975; Fruend, 1981; Forbush et al., 1982; Hoyt and Schatten, 1997; Devore, 2000; Badruddin and Singh, 2003a).

2.2. Significance test-procedure: t-test based

Let $X_1, X_2, X_3, \ldots, X_n$ be a sample with parameter $\mu$ and $\sigma^2$, then the variable has a $\chi^2$ probability distribution with $n - 1$ degrees of freedom, i.e.

$$\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{\sigma^2} = \frac{nS^2}{\sigma^2} \sim \chi^2_{n-1},$$

where $S^2$ is sample variance.
The square of standard normal distribution $N(0, 1)$, a $\chi^2$ distribution, with a subsample of $k$ values

$$\frac{nk(\bar{X}_k - \bar{X})^2}{\sigma^2(n-k)} \sim \chi^2_k = U^2 \text{ (say).} \quad (4)$$

From (3) and (4)

$$nS^2\frac{k(n-2)}{n-k} - nk(\bar{X}_k - \bar{X})^2 \sim \chi^2_{n-2} = Z \text{ (say)} \quad (5)$$

The $t$-distribution

$$t = \frac{U}{\sqrt{Z/n - 2}} \quad (6)$$

with $(n - 2)$ df, or

$$t = \frac{\bar{X}_k - \bar{X}}{S} \sqrt{\frac{k(n-2)}{n-k-k((\bar{X}_k - \bar{X})/S)^2}} \quad (7)$$

This expression represents the value of $t$-distribution with $(n - 2)$ df. It is different from that of Haung and Lee (1975) in the sense that here $S$ is sample standard deviation instead of sample variance. For a random sample of size $n$, whose mean is $\bar{X}$, from a distribution with mean $\mu$ and with same degree of freedom is given by

$$t = \frac{\bar{X} - \mu}{S/\sqrt{n-1}} \quad (8)$$

Then, the upper and lower confidence limit for $\bar{X}$ and $\bar{X}_k$, both with confidence level 100 $(1 - z)%$, with $(n - 2)$ df, are given by following expressions

$$\bar{X} \pm t_{a/2, n-2}S/\sqrt{n-1} \quad (9)$$

$$\bar{X}_k \pm t_{a/2, n-2}S\left\{\frac{k(n-2)}{n-k-k((\bar{X}_k - \bar{X})/S)^2}\right\}^{-1/2} \quad (10)$$

The significance of deviation from mean obtained by superposed epoch analysis can be tested using (9) and (10).

2.3. Significance test procedure: F-test based

Let $X_{ij}$ denotes the $j$th measurement from $i$th population, $J$ denotes the number of the observations in the each sample (epoch), and $I$ is the number of populations (epochs) being analyzed. The data set consists of $IJ$ observation. Let (individual) sample means (row average) be $\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_I$ and the column averages $\bar{X}_{j1}, \bar{X}_{j2}, \ldots, \bar{X}_{Jj}$ $(i = 1, 2, 3, \ldots, I; j = 1, 2, 3, \ldots, J)$.

To test the null hypothesis (i.e. all the means $\bar{X}_i$ are equal) two estimate of variance, one based on the variation among the $\bar{X}$’s and the other estimate of variance based on the variation within samples are compared (see also, Freund, 1981; Hoyt and Schatten, 1997; Devore, 2000; Badruddin and Singh, 2003b).

Let $S_{\bar{X}}^2$ be the estimate of variance based on the variation among the sample means. Then the extent to which the sample mean can be expected to fluctuate, or vary, due to chance i.e. the uncertainty in the determination of the mean called estimated error of the mean is

$$S_{\bar{X}} = S_{\bar{X}} \sqrt{J} \quad (11)$$

Variance of the sample mean of observation

$$S_{\bar{X}}^2 = \frac{S_1^2}{J} \quad (12)$$

e.i.

$$S_1^2 = JS_{\bar{X}}^2. \quad (13)$$

The variance based on the variation within the sample can be estimated from column variances $S_j^2$, $j = 1, 2, 3, \ldots, J$, by averaging them, i.e.

$$S_2^2 = \frac{\Sigma_{j=1}^J S_j^2}{J} \quad (14)$$

Now we have two estimate of variance, one from Eq. (13) and other from (14). We calculate

$$F = \frac{S_1^2}{S_2^2} \quad (15)$$

The null hypothesis is rejected if $F$ exceed $F_2$, the $F$-distribution to some $z$ level of significance. $F_2$ may be looked up in appropriate table. If we compare the means of $k$ samples of size $n$, we taken $k - 1$ degree of freedom for numerator and $k(n - 1)$ degree of freedom for the denominator.

3. Results

3.1. Transformation of data (removal of solar cycle effect): need and importance

Let us consider that the data for Chree analysis has a large superimposed variation other than the effect under study. For example, let one is studying the effect of certain solar/interplanetary phenomena on the transient modulation of cosmic ray intensity.
(Forbush decreases) spread over one or more solar cycles. The amplitude of Forbush decreases in cosmic ray intensity is usually $\sim 2 - 10\%$, while variation in cosmic ray intensity over a solar cycle may be $\sim 20 - 30\%$. To demonstrate the effect of solar cycle variation on the statistical analysis, we have selected Forbush decreases (due to interplanetary shocks) occurring during different phases of solar cycle. Three typical events selected from the total sample along with the average superposed epoch result has been shown in Fig. 1(a). Different ‘normal’ levels (before zero day) in three events are because of the solar cycle effect. In some case it is even more than the Forbush effect and the same effect is present, in respective events, in every day’s data. If a significance test is done without removing the solar cycle effect, the variation in the level of individual events will affect the results and it will not be the test of significance of ‘genuine’ effect. Therefore, before subjecting the data for significance test, solar cycle effect is removed as described in Section 2.1. It is done, in such a way that the mean of the sample, mean superposed values, as well as difference in counts between different days of an event, remain the same before and after the correction as shown in Fig. 1(a) and (b).

In order to demonstrate the need of data correction (removal of solar cycle effect), we have selected a number of Forbush decreases observed during the period of different solar activity level. Having carried out the Chree analysis we have applied the statistical procedure to test the significance of deviation in cosmic ray intensity from the average level before and after correction for solar cycle effect.

In Fig. 2 we have shown the mean superposed intensity for all the Forbush decreases considered, before correction (upper panel) and after correction (lower panel). It should be noted that the two profiles are exactly similar. The 95% confidence interval for the minimum intensity is calculated by using Eq. (10). It is shown by vertical line. The mean intensity for all days is shown by a (middle) horizontal line. The 95% confidence limits for mean intensity are calculated by using Eq. (9) and are shown by two (upper and lower) dotted horizontal lines, both in Figs. 2(a) and (b). If part of the vertical solid line representing the 95% confidence interval of decrease goes into the area bounded by the two dotted horizontal lines representing 95% confidence interval of average intensity, the observed effect (decrease) could have come about

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Fig. 1. Three typical FD events (square, circle, triangle) alongwith average profile (inverted triangle) before (Fig. 1(a)) and after (Fig. 1(b)) the correction for solar cycle effect.
simply through random fluctuations of the intensity (see also Huang and Lee, 1975). If, however, the vertical line, does not go across the lower horizontal line, it is very unlikely that the observed value of decrease is due to random fluctuations and we may suggest a physical reason between the source and subsequent decrease in cosmic ray intensity (effect).

If the test procedure is applied without correction for solar cycle effect (see Fig. 2(a)), one will infer that the observed effect (decrease) cannot be considered to be significant at 95% level, although all cases considered in this sample are clear Forbush decreases. However, when tested after correction for solar cycle effect, the results are found to be significant. This illustration suggests the need and importance of correction of data for ‘other’ major effects present in the data (e.g. solar cycle effect) before subjecting it to significance test for ‘genuine’ effect (e.g. Forbush effect).

3.2. Application and comparison of two techniques

Sudden commencement (SC) of geomagnetic storm signifies the arrival of interplanetary shock at earth (Cane, 1985). It is known that all the shocks arriving at earth do not produce observable effect in cosmic ray intensity recorded at earth (Cane, 2000; Kudela and Brenkus, 2004). Therefore, to study the effectiveness of shocks of different SC amplitude, on transient modulation of cosmic rays, and to illustrate the test of significance discussed in this paper, we have divided the shocks on the basis of SC amplitudes into three groups i.e. $5\gamma \leq SC \leq 25\gamma$ (Group I), $25\gamma < SC \leq 40\gamma$ (Group II) and $SC > 40\gamma$ (Group III). SC amplitude may be considered as an indicator of shocks strength. Superposed epoch analysis of neutron monitor data is then performed according to these three groups. These results are shown in Fig. 3. In the superposed epoch plot (Fig. 3) the decrease in cosmic ray intensity is seen due to all the three groups of shocks. If one does not perform test to determine the statistical reality of the observed decrease, following the arrival of shocks, one can infer, as has been done in most of the earlier works involving superposed epoch analysis, that all the three groups of shocks produce decreases in cosmic ray intensity. But, if one does a significance test, the conclusions may not be same.

To test the statistical significance of the ‘signal’, confidence limits are determined and shown in Fig. 4 for all the three groups before (left panel)
Fig. 3. Superposed epoch analysis results of cosmic ray intensity with respect to interplanetary shocks divided in three groups according to SC amplitude due to shocks; group I ($5 \gamma \leq SC \leq 25 \gamma$), group II ($25 \gamma < SC \leq 40 \gamma$), and group III ($SC > 40 \gamma$).

Fig. 4. Superposed epoch analysis results of cosmic ray intensity along with the standard error of mean bars before (left) and after (right) the correction, with respect to interplanetary shocks divided in three groups according to SC amplitude due to shocks; group I ($5 \gamma \leq SC \leq 25 \gamma$), group II ($25 \gamma < SC \leq 40 \gamma$), and group III ($SC > 40 \gamma$).
methods also allow one to make quantitative relationship between diverse data sets, statistical establishing the existence (or lack of existence) of a fluctuation in the data due to some other reason.

**4. Discussion**

In addition to providing a valuable method for establishing the existence (or lack of existence) of a relationship between diverse data sets, statistical methods also allow one to make quantitative assessments of the strengths of the observed relationships. One of the statistical methods, the superposed epoch analysis, has been employed very profitably for many solar-terrestrial, space and atmospheric physics problems using, for example, cosmic ray, ionospheric, heliospheric, atmospheric and meteorological data. Given proper physical insight in the choice of parameters and in the selection of zero-epoch time, the method is valuable in showing the average behavior of the system.

Although the method of superposed epoch (Chree) analysis has been utilized for several decades, a convenient procedure to determine the statistical significance of the results has not been available. Consequently, various subjective methods have been utilized in the interpretation of results. In some of the studies, error bars are given without mentioning how the bars are obtained.

Superposed epoch (Chree) analysis procedure is an important and widely used method for evaluating the statistical relationship between events of two different types. First application of the Chree or superposed epoch analysis reported a 27-day recurrence tendency in geomagnetic data (Chree, 1913). Later applications of this method confirmed the existence of 27-days period in magnetic storms (Chree and Stagg, 1927). Superposed epoch analysis by Greaves and Newton (1929) resulted in a breakthrough discovery (as remarked by Cliver, 1995, also see Crooker and Cliver, 1994) that the 27-day recurrence is primarily a property of the smaller geomagnetic storms and that larger storms occur sporadically. Several other subsequent superposed studies of magnetic storms/geomagnetic activity revealed the causes and effects of geomagnetic storms and many other features of geomagnetic activity. The 27-day recurrence tendency in cosmic ray data and sunspot data was revealed by use of Chree analysis (Mayer and Simpson, 1954). Forbush et al. (1982) have emphasized the importance of statistical test before reaching at a conclusion and have given a method for statistical test evaluating quasi-persitency in the data. In addition to revealing these periodicities in the activities on the solar surface (sunspot number), in the geomagnetsphere (geomagnetic data) and heliosphere (cosmic ray data), the superposed epoch analysis has also been used to demonstrate the effect of one parameter over the other. For example, effect of heliospheric current sheet passage on 10.7 cm solar flux (solar activity parameter), IMF magnitude (heliospheric parameter), Ap index (magnetospheric parameter),

<table>
<thead>
<tr>
<th>Group</th>
<th>Data</th>
<th>F</th>
<th>F-test results</th>
<th>t-test results</th>
</tr>
</thead>
<tbody>
<tr>
<td>I (SC ≤ 25γ)</td>
<td>Uncorrected 0.14</td>
<td>Insignificant</td>
<td>Insignificant</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Corrected 0.997</td>
<td>Insignificant</td>
<td>Insignificant</td>
<td></td>
</tr>
<tr>
<td>II (25γ &lt; SC ≤ 40γ)</td>
<td>Uncorrected 0.45</td>
<td>Insignificant</td>
<td>Insignificant</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Corrected 5.53</td>
<td>Significant</td>
<td>Significant</td>
<td></td>
</tr>
<tr>
<td>III (SC &gt; 40γ)</td>
<td>Uncorrected 2.34</td>
<td>Significant</td>
<td>Significant</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Corrected 10.24</td>
<td>Significant</td>
<td>Significant</td>
<td></td>
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The 95% confidence interval for minimum intensity (I_{min}), calculated using t-test based procedure is shown by solid vertical line. The mean intensity for all days (F) is shown by (middle) horizontal line. The 95% confidence limits, for F are shown by two dotted horizontal lines above and below the mean value. Values of F are also calculated using the method discussed in Section 2.3. Thus, both the tests for determination of confidence level have been performed to test for the statistical reality of the effects seen in cosmic ray intensity due to all the three group of shocks before and after correction for solar cycle effect. The 95% confidence interval of I_{min} and F plotted in left panel of Fig. 4 (before correction for solar cycle effect) suggest that effect of shocks due to Group I (5γ ≤ SC ≤ 25γ) and Group II (25γ < SC ≤ 40γ) cannot be treated as significant. However, the effects of Group III shocks (SC > 40γ) may be considered as significant. But, when the statistical tests were performed after the correction for solar cycle effect, shown in right panel of Fig. 4, the results of Group II shocks too is found to be significant. The results of F-test based significance limit also show similar results. Conclusions drawn after the two tests are tabulated in Table 1. Thus, we conclude that the effects due to shocks with Group II (25γ < SC ≤ 40γ) and Group III (SC > 40γ) shocks can be considered as significant and real while the effect seen due to Group I (SC ≤ 25γ) shocks cannot be treated as significant and may have come due to random fluctuations in the data due to some other reason.
galactic cosmic rays (in the heliosphere), radio wave absorption (in ionosphere), temperature at the 10-mb level (in atmosphere), vorticity area index and meteorological microseisms (climate/weather conditions) have been studied using Chree method of superposed epoch analysis (e.g. Lastovicka, 1979). But rarely statistical tests have been performed to see the significance of results.

In general, when dealing with the solar/heliospheric/magnetospheric/cosmic ray data, in addition to the one under study, there are other (superimposed) effects (e.g. solar cycle variation effect). Due to the effect other than the one under study, a significantly large variation in data set may also be observed.

The solar cycle effect, evident in Fig. 1(a) from different ‘normal’ levels (i.e. before zero day), has been removed from the data before it is subjected to tests. After the correction (transformation) for solar cycle effect, as shown in Fig. 1(b), individual events are nearly at the same (normal) level as against those shown in Fig. 1(a). This transformation is done without altering, (a) the grand mean, (b) the average value of each epoch (before and after zero day), (c) the magnitude of real ‘effect’ to be tested (e.g. Forbush effect) and (d) day-to-day variations in the data of each event. In Fig. 2(a) we have shown the statistical results showing 95% confidence interval of mean intensity and the decrease in intensity, without correcting the data for solar cycle effect. Although key days in the superposed epoch analysis correspond to shocks, all producing Forbush decreases, the test results appear to show that the observed decrease cannot be considered to be statistically significant. However, as shown in Fig. 2(b) when the statistical test was done after correcting data as described in Section 2.1, the observed intensity decrease is statistically significant.

The effects of solar flares, high speed solar wind streams, interplanetary shocks, CMEs and/or magnetic clouds, on solar wind parameters in the heliosphere (proton temperature, density etc.) geomagnetic parameter (Ap, Kp, Dst etc.), charge particle flux (cosmic ray counts), ionospheric behavior (total electron content) has been studied by using Chree analysis. Influence of sudden changes in cosmic ray intensity (Forbush decreases) or tropospheric conductivity, cloud cover, atmospheric circulation, ionospheric absorption, rainfall, sunshine data, pressure level heights, temperature profile and wind characteristics, vorticity area index, solar radiation input in the lower atmosphere, atmospheric electricity and air earth current density, total ozone content etc. have been studied by employing this technique. But a proper test of significance is not done in most of these studies for arriving at a definite conclusion.

5. Conclusions

Two statistical procedures, one based on \( t \)-test and other based on \( F \)-test, to access the statistical reality of an average variation as obtained from superposed epoch analysis is described. A procedure to transfer (correct) the data for other (superimposed) real effect under study is given. This procedure is useful for the space data, in general, and cosmic ray data, in particular, where other effect (e.g. solar cycle effect) of a large magnitude is present in the data sets in addition to one under study. The importance of this correction has also been demonstrated, using cosmic ray data, as an example. It is suggested that data having other superimposed effect of comparable amplitude, should be corrected first for such effects, before subjecting it to the test of significance for genuine signal.

Two statistical procedures developed to test the significance of the results obtained by Chree analysis have been applied and effectiveness of the shocks, with different SC amplitude, on cosmic ray intensity is tested. Both the techniques lead to similar conclusions. Both statistical tests demonstrate that effect observed in cosmic ray variation, due to shocks with \( SC \leq 25 \gamma \), is not significant at 95% confidence level. At the same level of confidence, these tests lead to conclusion that the decrease in intensity due to shocks with \( 25 \gamma < SC \leq 40 \gamma \) and \( SC > 40 \gamma \) is significant and does not come due to random fluctuations of sampling.

It is suggested that before reaching at the conclusion about the effect of one phenomenon over the other, based on Chree analysis, the significance of the observed variations should better be tested.

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